

Gravitational models with non-local scalar fields

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Abstract

A general class of $f(R)$ gravity models with minimally coupling a nonlocal scalar field is considered. The Ostrogradski representation for nonlocal gravitational models with a quadratic potential and the way of its localization are proposed. We study the action with an arbitrary analytic function $\mathcal{F}(\Box_g)$, which has both simple and double roots. The way of localization allows to find particular solutions of nonlocal equations of gravity.

1 Introduction

Recently a new class of cosmological models based on the string field theory (SFT) [1] and the p -adic string theory emerges and attracts a lot of attention [2]–[20]. It is known that the SFT and the p -adic string theory are UV-complete ones. Thus, one can expect that resulting (effective) models should be free of pathologies. These models exhibit one general non-standard property, namely, their actions have terms with infinitely many derivatives, i.e. nonlocal terms. The higher derivative terms usually produce phantom fields [21, 22] (see also [23]). Models that includes phantoms violate the null energy condition (NEC), and, therefore, are unstable. Models with higher derivative terms produce also well-known problems with quantum instability [23].

To obtain a stable model with the NEC violation (the state parameter $w_{\text{DE}} < -1$) one should construct this model as an effective model, connected with the fundamental theory, which is stable and admits quantization. With the lack of quantum gravity, we can just trust string theory or deal with an effective theory admitting the UV completion.

The purpose of this paper is to study $f(R)$ gravity models with a nonlocal scalar field. We consider a general form of nonlocal action for the scalar field with a quadratic potential, keeping the main ingredient, the analytic function $\mathcal{F}(\Box_g)$, which in fact produces the nonlocality, almost unrestricted.

2 Nonlocal gravitation models

The SFT inspired nonlocal gravitation models [2] are introduced as a sum of the SFT action of the tachyon field ϕ plus the gravity part of the action. One cannot deduce this form of the action

from the SFT. In this paper we study the $f(R)$ gravity, which is a straightforward modification of the general relativity. We consider the following action:

$$S_f = \int d^4x \sqrt{-g} \left(\frac{f(L^2 R)}{16\pi G_N L^2} + \frac{1}{\alpha' g_o^2} \left(\frac{1}{2} \phi \mathcal{F}(\alpha' \square_g) \phi - V(\phi) \right) - \Lambda \right), \quad (1)$$

where $f(L^2 R)$ is an arbitrary differentiable function. We use the signature $(-, +, +, +)$, $g_{\mu\nu}$ is the metric tensor, G_N is the Newtonian constant. The potential $V(\phi)$ is a quadratic polynomial $V(\phi) = C_2 \phi^2 + C_1 \phi + C_0$, where C_2 , C_1 , and C_0 are arbitrary real constants.

The function \mathcal{F} is assumed to be analytic at all finite points of the complex plane, in other words, to be an entire function. The function \mathcal{F} can be represented by the convergent series expansion: $\mathcal{F}(\square_g) = \sum_{n=0}^{\infty} f_n \square_g^n$. The Weierstrass factorization theorem asserts that the function \mathcal{F} can be represented by a product involving its zeroes J_k :

$$\mathcal{F}(J) = J^m e^{Y(J)} \prod_{k=1}^{\infty} \left(1 - \frac{J}{J_k} \right) e^{\frac{J}{J_k} + \frac{J^2}{2J_k^2} + \dots + \frac{1}{p_k} \left(\frac{J}{J_k} \right)^{p_k}}, \quad (2)$$

where m is an order of the root $J = 0$ (m can be equal to zero), $Y(J)$ is an entire function, natural numbers p_n are chosen such that the series $\sum_{n=1}^{\infty} \left(\frac{J}{J_n} \right)^{p_n+1}$ is an absolutely and uniformly convergent one.

Scalar fields ϕ (associated with the open string tachyon) is dimensionless, while $[\alpha'] = \text{length}^2$, $[L] = \text{length}$ and $[g_o] = \text{length}$. Let us introduce dimensionless coordinates $\bar{x}_\mu = x_\mu / \sqrt{\alpha'}$, the dimensionless Newtonian constant $\bar{G}_N = G_N / \alpha'$, the dimensionless parameter $\bar{L} = L / \sqrt{\alpha'}$, and the dimensionless open string coupling constant $\bar{g}_o = g_o / \sqrt{\alpha'}$. The dimensionless cosmological constant $\bar{\Lambda} = \Lambda \alpha'^2$, \bar{R} is the curvature scalar in the coordinates \bar{x}_μ :

$$S_f = \int d^4\bar{x} \sqrt{-g} \left(\frac{f(\bar{L}^2 \bar{R})}{16\pi \bar{G}_N \bar{L}^2} + \frac{1}{\bar{g}_o^2} \left(\frac{1}{2} \phi \mathcal{F}(\bar{\square}_g) \phi - V(\phi) \right) - \bar{\Lambda} \right). \quad (3)$$

In the following formulae we omit bars, but use only dimensionless coordinates and parameters.

It is well-known [24] that at $f'(R) > 0$ any $f(R)$ gravity models in the metric variational approach are equivalent to the Einstein gravity with a scalar field¹. In the metric variational approach the equations of gravity are as follows:

$$G_{\mu\nu} \equiv f'(R) R_{\mu\nu} - \frac{f(R)}{2} g_{\mu\nu} - D_\mu \partial_\nu f'(R) + g_{\mu\nu} \square_g f'(R) = 8\pi G_N T_{\mu\nu}, \quad \mathcal{F}(\square_g) \phi = \frac{dV}{d\phi}, \quad (4)$$

where the energy-momentum (stress) tensor $T_{\mu\nu}$ is:

$$T_{\mu\nu} \equiv - \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{g_o^2} \left(E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu} (g^{\rho\sigma} E_{\rho\sigma} + W) \right), \quad (5)$$

$$E_{\mu\nu} \equiv \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu \square_g^l \phi \partial_\nu \square_g^{n-1-l} \phi, \quad W \equiv \frac{1}{2} \sum_{n=2}^{\infty} f_n \sum_{l=1}^{n-1} \square_g^l \phi \square_g^{n-l} \phi - \frac{f_0}{2} \phi^2 + C_1 \phi. \quad (6)$$

¹There are two types of $f(R)$ gravity: the metric variational approach and the Palatini formalism. In the first case the equations of motion are obtained by variation with respect to metric. Connections are the function of metric in this formalism. In the Palatini formalism one should vary the action independently with respect to metric and the connections.

3 Localization of nonlocal gravitational actions

The Ostrogradski representation has been proposed for polynomial $\mathcal{F}(\square)$ in the Minkowski space-time [21, 22]. Our goal is to generalize this result on gravitational models with an arbitrary analytic function $\mathcal{F}(\square)$ with simple and double roots. We also generalize the Ostrogradski representation on the models with a linear potential. The nonlocal cosmological models with quadratic potentials have been studied in [6, 7, 13, 14, 15, 16, 18].

Let us start with the case $C_1 = 0$. We consider a function $\mathcal{F}(J)$, which has simple roots J_i and double roots \tilde{J}_k , and the function

$$\phi_0 = \sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k, \quad (7)$$

where

$$(\square_g - J_i)\phi_i = 0 \quad \text{and} \quad (\square_g - \tilde{J}_k)^2 \tilde{\phi}_k = 0 \quad \Leftrightarrow \quad (\square_g - \tilde{J}_k)\tilde{\phi}_k = \varphi_k, \quad (\square_g - \tilde{J}_k)\varphi_k = 0. \quad (8)$$

Without loss of generality we assume that for any i_1 and $i_2 \neq i_1$ conditions $J_{i_1} \neq J_{i_2}$ and $\tilde{J}_{i_1} \neq \tilde{J}_{i_2}$ are satisfied.

The energy-momentum tensor, which corresponds to ϕ_0 , has the following form:

$$T_{\mu\nu}(\phi_0) = T_{\mu\nu}\left(\sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k\right) = \sum_{i=1}^{N_1} T_{\mu\nu}(\phi_i) + \sum_{k=1}^{N_2} T_{\mu\nu}(\tilde{\phi}_k), \quad (9)$$

where all $T_{\mu\nu}$ are given by (5) and

$$E_{\mu\nu}(\phi_i) = \frac{\mathcal{F}'(J_i)}{2} \partial_\mu \phi_i \partial_\nu \phi_i, \quad E_{\mu\nu}(\tilde{\phi}_k) = \frac{\mathcal{F}''(\tilde{J}_k)}{4} \left(\partial_\mu \tilde{\phi}_k \partial_\nu \varphi_k + \partial_\nu \tilde{\phi}_k \partial_\mu \varphi_k \right) + \frac{\mathcal{F}'''(\tilde{J}_k)}{12} \partial_\mu \varphi_k \partial_\nu \varphi_k, \quad (10)$$

$$W(\phi_i) = \frac{J_i \mathcal{F}'(J_i)}{2} \phi_i^2, \quad W(\tilde{\phi}_k) = \frac{\tilde{J}_k \mathcal{F}''(\tilde{J}_k)}{2} \tilde{\phi}_k \varphi_k + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right) \varphi_k^2, \quad (11)$$

where a prime denotes a derivative with respect to J : $\mathcal{F}' \equiv \frac{d\mathcal{F}}{dJ}$, $\mathcal{F}'' \equiv \frac{d^2\mathcal{F}}{dJ^2}$ and $\mathcal{F}''' \equiv \frac{d^3\mathcal{F}}{dJ^3}$.

Considering the following local action

$$S_{loc} = \int d^4x \sqrt{-g} \left(\frac{f(R)}{16\pi G_N} - \Lambda \right) + \sum_{i=1}^{N_1} S_i + \sum_{k=1}^{N_2} \tilde{S}_k, \quad (12)$$

where

$$S_i = -\frac{1}{g_o^2} \int d^4x \sqrt{-g} \frac{\mathcal{F}'(J_i)}{2} (g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + J_i \phi_i^2), \quad (13)$$

$$\begin{aligned} \tilde{S}_k = & -\frac{1}{g_o^2} \int d^4x \sqrt{-g} \left(g^{\mu\nu} \left(\frac{\mathcal{F}''(\tilde{J}_k)}{4} \left(\partial_\mu \tilde{\phi}_k \partial_\nu \varphi_k + \partial_\nu \tilde{\phi}_k \partial_\mu \varphi_k \right) + \right. \right. \\ & \left. \left. + \frac{\mathcal{F}'''(\tilde{J}_k)}{12} \partial_\mu \varphi_k \partial_\nu \varphi_k \right) + \frac{\tilde{J}_k \mathcal{F}''(\tilde{J}_k)}{2} \tilde{\phi}_k \varphi_k + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right) \varphi_k^2 \right), \end{aligned} \quad (14)$$

we can see that solutions of the Einstein equations and equations in ϕ_k , $\tilde{\phi}_k$ and φ_k , obtained from this action, solve the initial nonlocal equations (4). Thus, we obtain that special solutions to nonlocal equations can be found as solutions to system of local (differential) equations. If $\mathcal{F}(J)$ has an infinity number of roots then one nonlocal model corresponds to infinity number of different local models and the initial nonlocal action (3) generates infinity number of local actions (12).

We should prove that the way of localization is self-consistent. To construct local action (12) we assume that equations (8) are satisfied. Therefore, the method of localization is correct only if these equations can be obtained from the local action S_{loc} . The straightforward calculations show that the way of localization is self-consistent because:

$$\frac{\delta S_{loc}}{\delta \phi_i} = 0 \Leftrightarrow \square_g \phi_i = J_i \phi_i; \quad \frac{\delta S_{loc}}{\delta \tilde{\phi}_k} = 0 \Leftrightarrow \square_g \varphi_k = \tilde{J}_k \varphi_k; \quad \frac{\delta S_{loc}}{\delta \varphi_k} = 0 \Leftrightarrow \square_g \tilde{\phi}_k = \tilde{J}_k \tilde{\phi}_k + \varphi_k. \quad (15)$$

In spite of the above-mention equations we obtain from S_{loc} the equations:

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu}(\phi_0) - \Lambda g_{\mu\nu}), \quad (16)$$

where ϕ_0 is given by (7) and $T_{\mu\nu}(\phi_0)$ can be calculated by (9). So, we get such systems of differential equations that any solutions of these systems are particular solutions of the initial nonlocal equations (4).

Let us consider functions $\mathcal{F}(J)$ with two and only two simple roots. If $\mathcal{F}(J)$ has two real simple roots, then $\mathcal{F}'(J) > 0$ at one root and $\mathcal{F}'(J) < 0$ at another root, so we get a quintom model [25], in other words, local model with one standard scalar field and one phantom scalar field. In the case of two complex conjugated simple roots J_j and J_j^* one gets the following action:

$$S_c = \int d^4x \frac{\sqrt{-g}}{2g_o^2} (\mathcal{F}'(J_j) (g^{\mu\nu} \partial_\mu \phi_j \partial_\nu \phi_j + J_j \phi_j^2) + \mathcal{F}'^*(J_j) (g^{\mu\nu} \partial_\mu \phi_j^* \partial_\nu \phi_j^* + J_j^* \phi_j^{*2})). \quad (17)$$

We introduce real fields ξ and η such that $\phi_j = \xi + i\eta$, $\phi_j^* = \xi - i\eta$, denote $d_r \equiv \Re e(\mathcal{F}'(J))$, $d_i \equiv \Im m(\mathcal{F}'(J))$, and obtain:

$$S_c = \int d^4x \frac{\sqrt{-g}}{2g_o^2} (d_r g^{\mu\nu} (\partial_\mu \xi \partial_\nu \xi - \partial_\mu \eta \partial_\nu \eta) + d_i g^{\mu\nu} (\partial_\mu \xi \partial_\nu \eta - \partial_\mu \eta \partial_\nu \xi) + V_1), \quad (18)$$

where V_1 is a potential term. In the case $d_i = 0$ we get a quintom model, in opposite case the kinetic term in S_c has a nondiagonal form. To diagonalize the kinetic term we make the transformation: $\chi = v + \tilde{C}\sigma$, $\eta = -\tilde{C}v + \sigma$, where $\tilde{C} \equiv (d_r + \sqrt{d_r^2 + d_i^2})/d_i$, and get a quintom model:

$$S_c = \int d^4x \frac{\sqrt{-g}}{2g_o^2} \left(\frac{2(d_r^2 + d_i^2)}{d_i^2} \left(d_r + \sqrt{d_r^2 + d_i^2} \right) (\partial_\mu v \partial_\nu v - \partial_\mu \sigma \partial_\nu \sigma) + V_1 \right). \quad (19)$$

In the case of a real double root \tilde{J}_k we express $\tilde{\phi}_k$ and φ_k in terms of new fields ξ_k and χ_k :

$$\tilde{\phi}_k = \frac{1}{2\mathcal{F}''(\tilde{J}_k)} \left(\left(\mathcal{F}''(\tilde{J}_k) - \frac{2}{3}\mathcal{F}'''(\tilde{J}_k) \right) \xi_k - \left(\mathcal{F}''(\tilde{J}_k) + \frac{2}{3}\mathcal{F}'''(\tilde{J}_k) \right) \chi_k \right), \quad \varphi_k = \xi_k + \chi_k,$$

we obtain the corresponding \tilde{S}_k in the following form:

$$\begin{aligned}\tilde{S}_k = & \frac{-1}{2g_o^2} \int d^4x \sqrt{-g} \left(g^{\mu\nu} \frac{\mathcal{F}''(\tilde{J}_k)}{4} (\partial_\mu \xi_k \partial_\nu \xi_k - \partial_\nu \chi_k \partial_\mu \chi_k) + \left[\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right] (\xi_k + \chi_k)^2 + \right. \\ & \left. + \frac{\tilde{J}_k}{4} \left[(\mathcal{F}''(\tilde{J}_k) - \frac{2}{3} \mathcal{F}'''(\tilde{J}_k)) \xi_k - (\mathcal{F}''(\tilde{J}_k) + \frac{2}{3} \mathcal{F}'''(\tilde{J}_k)) \chi_k \right] (\xi_k + \chi_k) \right).\end{aligned}$$

It is easy to see that each \tilde{S}_k includes one phantom scalar field and one standard scalar field. So, in the case of one double root we obtain a quintom model. In the Minkowski space appearance of phantom fields in models, when $\mathcal{F}(J)$ has a double root, has been obtained in [22]. So, we come to conclusion that both two simple roots and one double root of $\mathcal{F}(J)$ generate quintom models.

The model with action (3) in the case $C_1 \neq 0$ has been considered in detail in [18]. Here we present only the obtained algorithm of localization for an arbitrary quadratic potential $V(\phi) = C_2\phi^2 + C_1\phi + C_0$:

- Change values of f_0 and Λ such that the potential takes the form $V(\phi) = C_1\phi$.
- Find roots of the function $\mathcal{F}(J)$ and calculate orders of them. Select an finite number of simple and double roots.
- Construct the corresponding local action. In the case $C_1 = 0$ one should use formula (12). In the case $C_1 \neq 0$ and $f_0 \neq 0$ one should use (12) with the replacement of the scalar field ϕ by χ and the corresponding modification of the cosmological constant. In the case $C_1 \neq 0$ and $f_0 = 0$ the local action is the sum of (12) and either

$$S_\psi = -\frac{1}{2g_o^2} \int d^4x \sqrt{-g} \left(f_1 g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + 2C_1 \psi + \frac{f_2 C_1^2}{f_1^2} \right), \quad (20)$$

in the case of simple root $J = 0$, or

$$S_{\tilde{\psi}} = -\int d^4x \frac{\sqrt{-g}}{2g_o^2} \left[g^{\mu\nu} \left(f_2 (\partial_\mu \tilde{\psi} \partial_\nu \tau + \partial_\nu \tilde{\psi} \partial_\mu \tau) + f_3 \partial_\mu \tau \partial_\nu \tau \right) + f_2 \tau^2 + 2C_1 \tilde{\psi} + \frac{f_3 C_1}{2f_2} \tau \right]$$

in the case of double root $J = 0$. Note that in the case $C_1 \neq 0$ and $f_0 = 0$ the local action (12) has no term, which corresponds to the root $J = 0$.

- Vary the obtained local action and get a system of the Einstein equations and equations of motion. The obtained system is a finite order system of differential equations, *i.e.* we get a local system. Seek solutions of the obtained local system.

4 Conclusion

The main result of this paper is the generalization of the algorithm of localization on the $f(R)$ gravity models with a nonlocal scalar field. The algorithm of localization is proposed for an

arbitrary analytic function $\mathcal{F}(\square_g)$, which has both simple and double roots. We have proved that the same functions solve the initial nonlocal Einstein equations and the obtained local Einstein equations. We have found the corresponding local actions and proved the self-consistence of our approach. In the case of two simple roots as well as in the case of one double root we get a quintom model [25]. The algorithm of localization does not depend on metric, so it can be used to find solutions for any metric.

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